LAPLACE'S EQUATION

If an enclosed surface does not have any volume charge ρ_v, then the poisson's equation becomes

$$\boxed{\nabla^2 V = 0} \quad \because \rho_v = 0$$

This is called as Laplace equation

[$\nabla^2 \rightarrow$ Laplacian operator]

$\rightarrow$ The solutions of Laplace's equation are the harmonic function

$\rightarrow$ Harmonic functions are important in partial differential equation theory, physics and engineering field.

Application's $\rightarrow$

It is used in boundary value problems.

'E' is the gradient of electric potential in opposite direction

* Gauss's law in point form

$$\boxed{\nabla \cdot D = \rho_v} \quad \text{——} \; ②$$

The divergence of electric flux density is equal to the volume charge density.

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot (\varepsilon E) = \rho_v \qquad (\because D = \varepsilon E$$

$$\varepsilon (\nabla \cdot E) = \rho_v$$

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon} \quad \text{——} \; ③$$

Sub. eq^n ① $E = -\nabla V$ in ③

$$\nabla \cdot (-\nabla V) = \frac{\rho_v}{\varepsilon}$$

$$-\nabla \cdot \nabla V = \frac{\rho_v}{\varepsilon}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$

This is called as Poisson's equation.

For cartesian co-ordinate system (x, y, z)

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}$$

For cylindrical co-ordinate system (ρ, ϕ, z)

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{dV}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}$$

For Spherical co-ordinate system (r, θ, ϕ)

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right)$$

$$+ \frac{1}{r^2 \sin^2\theta} \cdot \frac{\partial^2 V}{\partial \phi^2} = -\frac{\rho_v}{\varepsilon}$$

Poisson's equation →

Poisson's equation relates the electric potential to charge density

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$

where V = Electric Potential
ρ_v = volume charge density
ε = dielectric constant

The potential of a conductor is proportional to the charge it contains.

Proof →

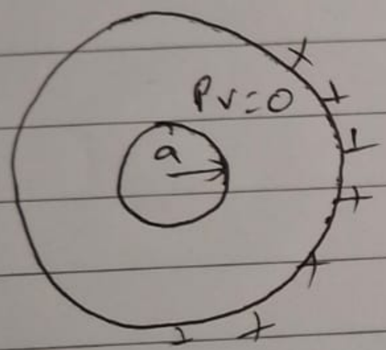
The relation between electric potential and field intensity is given as

$$\boxed{E = -\nabla V} \quad \text{— (1)}$$

In such cases, the charge distribution is involved on the surface of the conductor for which the volume charge density is zero.

Ex:—

To obtain potential and capacitance in between two surface.



Potential between conductors of a coaxial cable.

— x —